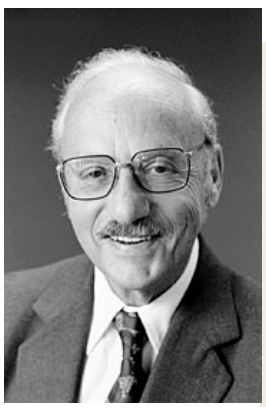
10 things you need to know

Simplex Method

http://www.linearprogramming.info/tag/simplex-method/



1. History

The Simplex Method is an algorithm which allows us to solve linear programming models (LP)

In 1947 it was first published by George Dantzig, an American mathematician.

Its first computing implementation was in 1952 to solve a problem with 71 variables and 48 equations. Solving it takes 18 hours.

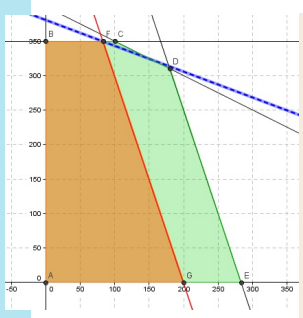
$$\text{Min } c^T x \quad \text{s.a } Ax = b \quad x \geq 0$$

2. Standard Form

The Simplex Method's application requires the linear programming model to be on its standard form.

We can assume without loss of generality that a linear programming model results from the standard form.

To do this we can add nonnegative slack variables, auxiliary variables or excess variables if needed.

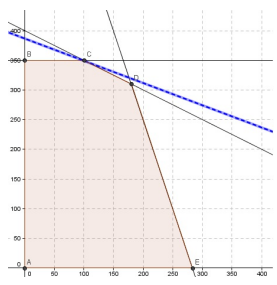


3. Basic Solution

The Simplex Method searches for the optimum solution to a linear problem through the vertices in domain of feasible solutions.

A basic feasible solution matches a vertices in domain of a Linear Programming model's feasible solutions

A basic feasible solution satisfies the standard form's conditions and the decision variables are nonnegative.

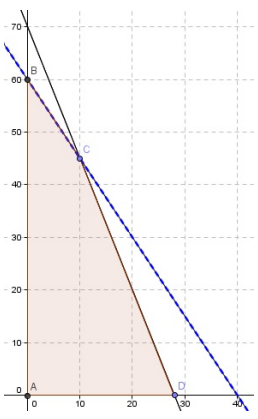


4. Optimality Criterion

The Basic feasible solution is optimal if and only if the reduced costs of all the nonbasic variables are higher or equal to zero.

X	Y	S1	S2	S3	
1	0	1/2	0	-2	100
0	0	-3	1	10	400
0	1	0	0	1	350
0	**0**	**3/2**	**0**	**2**	**3.100**

In this example the optimal basic feasible solution is X=100, Y=350 and S2=400. The nonbasic variables S1 and S3 have nonnegative reduced costs.

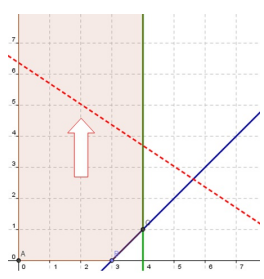


5. Infinite Solutions

This case is seen when there are reduced costs equal to zero in one or more optimal nonbasic variables.

X	Y	S1	S2	
0	1	2/5	-1/5	8/5
1	0	-3/10	2/5	14/5
0	**0**	**1/2**	**0**	**6**

In this example we observe an optimal basic feasible solution, where the nonbasic variable S2 has a reduced cost equal to zero.

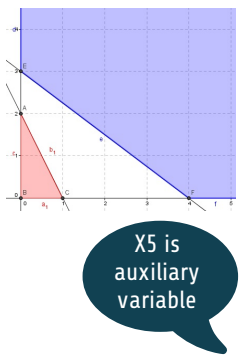


6. Unbounded Problem

This case is seen when, while doing the calculus of the variable which leaves the base, all the elements ykj of the column j in the tableau are negative.

X1	X2	X3	X4	
2	-2	1	0	6
4	0	0	1	16
-4	**-6**	**0**	**0**	**0**

Where j matches the index of a nonbasic variable with negative reduced cost.



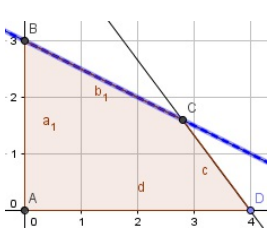
7. Infeasible Problem

This is the case when the optimal value of the problem in Phase 1 (in a Simplex Method of Two Phases) is nonzero.

X5 is auxiliary variable

x1	x2	x3	x4	x5	
2	1	1	0	0	2
-5	0	-4	-1	1	4
5	0	4	0	0	-4

A Linear Programming model is infeasible when the feasible solutions domain is empty.



8. Rate of Convergence

If two or more nonbasic variables have negative reduced cost, the entrance to the base goes to the lowest reduced cost.

Y enters the base

X	Y	S1	S2	
2	4	1	0	12
4	3	0	1	16
-1	**-2**	**0**	**0**	**0**

This way we try to get a lower number of iterations to achieve the optimal solution of the LP.

$$\text{Min } c^T x \quad \text{s.a. } Ax = b \quad x \geq 0$$

I	$B^{-1}D$	$B^{-1}b$
0	$C_D^T - C_B^T B^{-1} D$	$-C_B^T B^{-1} b$

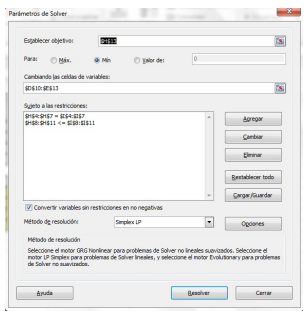
9. Postoptimal Analysis

Once we achieve the optimal solution through the Simplex Method, we can analyze the impact in the results with the alteration of the parameters

Frequent Analysis are:

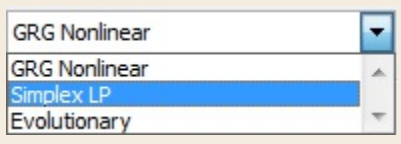
Modification on the "right hand side" Modification on the coefficient of the objective function. Add a new variable Add a new restriction

he goal is to analyze the possible changes in the optima olution and the optimal value of the model with no nee to re-optimize.



10. Excel Solver

The Excel complement "Solver" allows us to use the Simplex Method as a solving method (Simplex LP)



Simplex PL turns out to be specially suitable for the resolution of Liner Programming models.

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